Grade 6 Math Circles
February 28 March 1/2, 2023
BCC and Gauss Prep

Beaver Computing Challenge

The Beaver Computing Challenge (BCC) is an online problem-solving contest with a focus on computational and logical thinking. No prior coding experience is required. The questions are inspired by topics in computer science but students only require the concepts taught in the mathematics curriculum common to all provinces.

Students in grade 6 or below can write the Grade 5/6 BCC. The Grade 5/6 BCC consists of 12 multiple choice questions divided into 3 parts with 60 marks total: 4 questions in Part A worth 6 marks each, 4 questions in Part B worth 5 marks each, and 4 questions in Part C worth 4 marks each. Students are given exactly 45 minutes to answer the questions. Some calculators are permitted.

Each question on the BCC is given by a story and a question. The story provides the background information required to solve the question.

Resources
More information on the BCC: https://cemc.uwaterloo.ca/contests/bcc.html
Past contests/solutions: https://www.cemc.uwaterloo.ca/contests/past_contests.html#bcc

Strategies

Solving the BCC problems requires computational and logical thinking. Listed here are a few strategies to approach these types of problems.

- First read the story, then the question, then reread the story. This will help with finding the details needed to solve the question as well as understanding what the question is asking.
- Underline or write down the important information in the story and question.
- If the question is long and/or challenging, split it into pieces or steps. Focus on one step at a time, then connect them all together at the end.

- Make a chart or diagram to help organize what is given in the story. Or create another image that will help to visualize what is happening in the problem.
- Rule out answers that are impossible or that you can show aren't the solution. The problems are all multiple choice and will have 4 options, so ruling out a couple incorrect answers can help with deciding on the correct answer. When in doubt, make a logical guess.
- Have fun writing the contest! This contest is meant to be an enjoyable experience that will motivate your interest in math and computer science. The BCC emphasizes participation rather than competition, so be proud of trying.
- Following the end of the contest window, solutions will be posted on the website at this link: https://www.cemc.uwaterloo.ca/contests/past_contests.html#bcc

Past BCC Problems

Bird House

A parent takes their child to the store to buy a bird house. The child says "I would like a bird house with two windows and a heart decoration."

Which of the bird houses matches this description?

(A) House 1

(B) House 2

(C) House 3

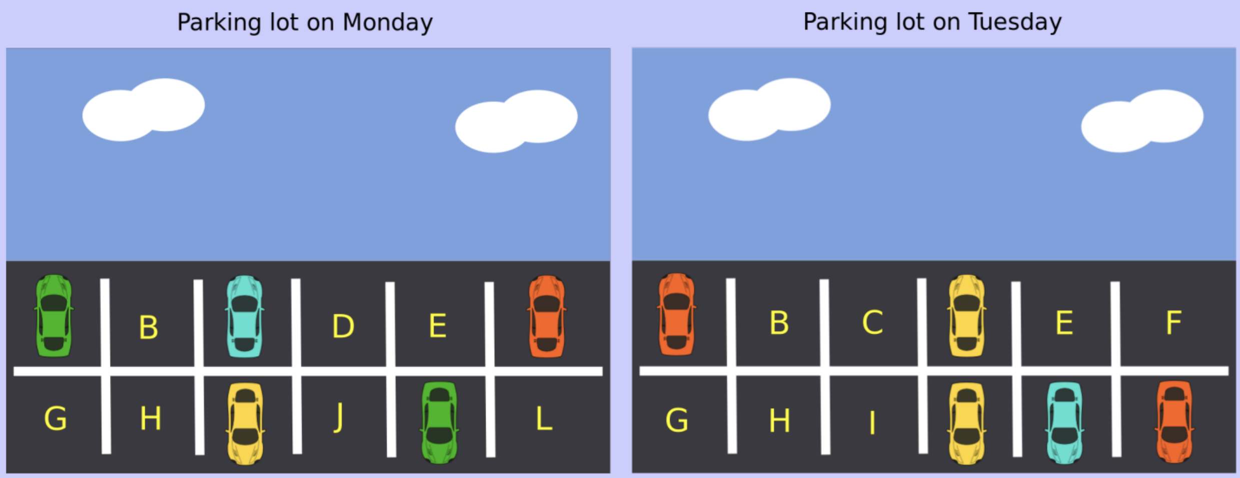
(D) House 4

Solution

Only House 2 and 3 have two windows, so we can immediately eliminate House 1 and 4 from our selection. Between House 2 and 3, only House 3 has a heart decoration. So the answer must be (C).

Parking Lot

There are 12 spaces for cars in a parking lot. The pictures below show which spaces were used on Monday and which spaces were used on Tuesday.



How many parking spaces were empty on both Monday and Tuesday?

(A) 3

(B) 4

(C) 5

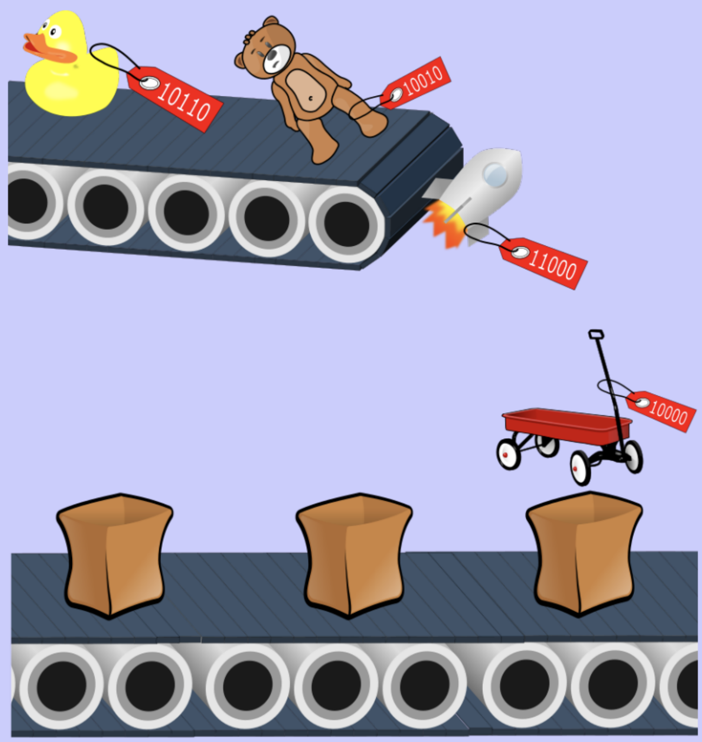
(D) 6

Solution

To find the answer, we count the parking spots that are in both pictures. Only parking spots B, E, G, and H appear in both pictures, so the answer must be (B).

Toy Factory

Toys fall from a high conveyor belt into bags on a low conveyor belt. The toys should fall into the bags with their numerical product codes in increasing order. One of the toys is in the wrong place and needs to be removed so that the remaining toys are in the right order.



Which toy must be removed?

(A) Cart

(B) Toy rocket

(C) Teddy bear

(D) Rubber duck

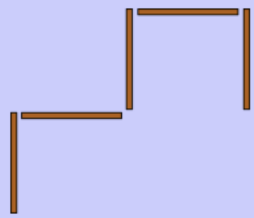
Solution

The tag says 10000 for the cart, 11000 for the toy rocket, 10010 for the teddy bear, and 10110 for the rubber duck. Since we want the toys to fall in increasing order, the first toy should have the smallest number on its tag while the last toy should have the largest number on its tag.

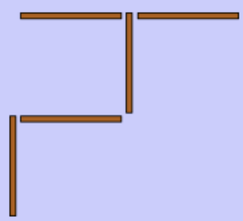
The largest tag is 11000 since the first two digits on the left are 11 while the first two digits on the left for every other tag are 10. So the toy rocket is out of place, and the answer must be (B).

5 Sticks

Nola puts five sticks on the table making this shape:

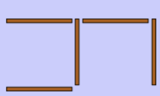
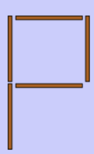
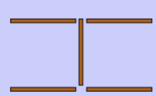


Then Adam moves one stick to a different place making this shape:



Now, Vera wants to move one stick to a different place.

Which shape is Vera **not** able to make?

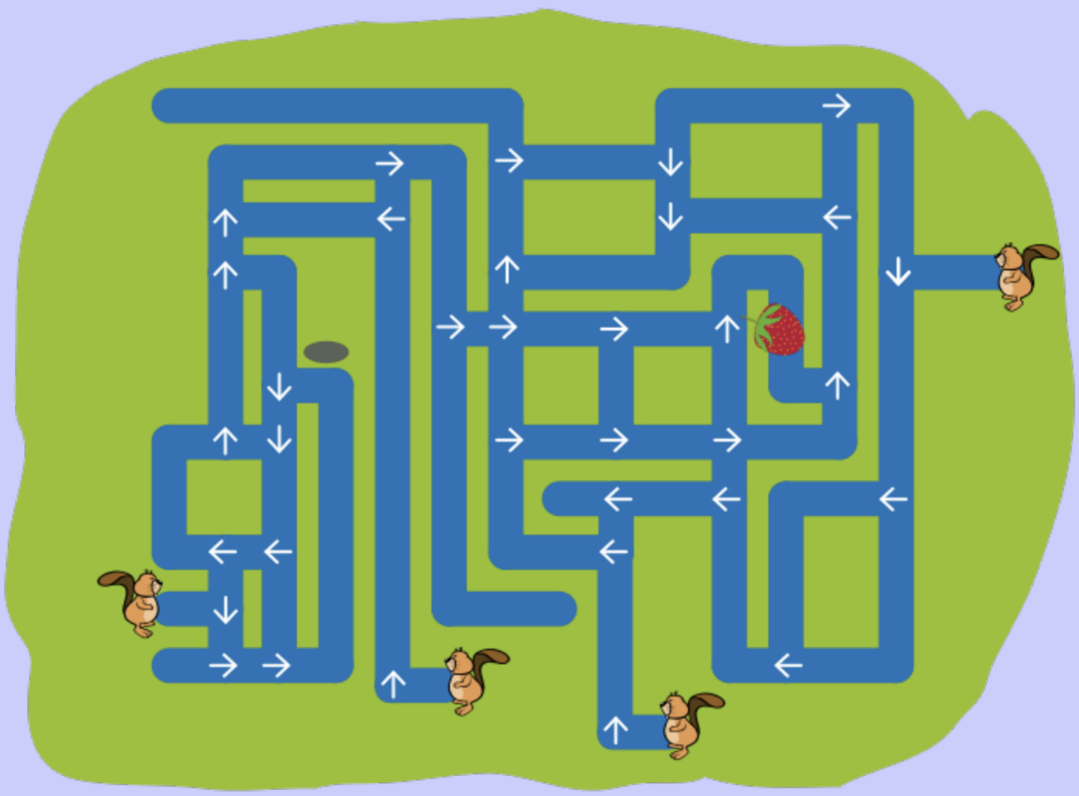
(A)  (B)  (C)  (D) 

Solution

Vera can make (A) by moving the bottom left stick, make (B) by moving the top right stick, and make (C) by moving the bottom left stick again. So the answer is (D) by trial and error.

Strawberry Hunt

Four beavers swim through canals in an attempt to find a strawberry. They start at different places and always move in the direction of the arrows shown below.



Each beaver either finds the strawberry, swims in a loop forever, or reaches and remains at a dead-end.

How many beavers find the strawberry?

(A) 1

(B) 2

(C) 3

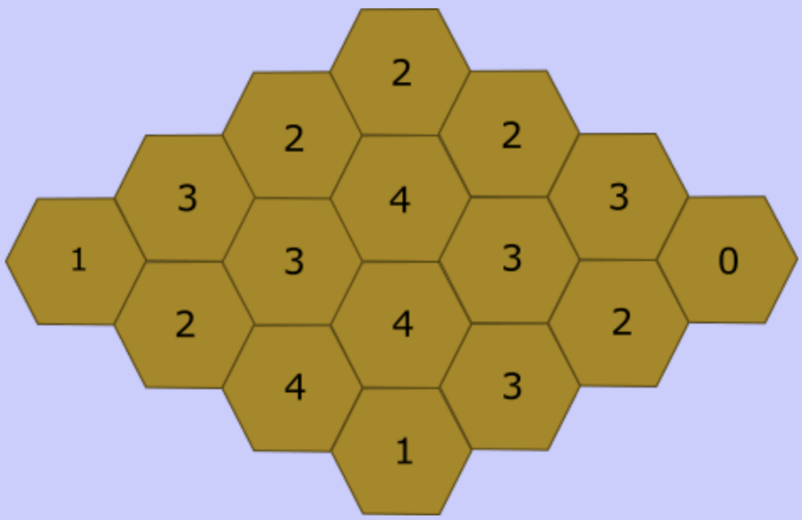
(D) 4

Solution

Tracing all the beavers, we see that the first two beavers (from left to right) are able to find the strawberry. The third beaver gets stuck in a loop, and the fourth beaver gets stuck in a dead end. So the answer must be (B).

Beehive

A bear studies how many hexagons in a honeycomb contain honey. For each hexagon, the bear records how many other hexagons touching this hexagon contain honey. So this number could be 0, 1, 2, 3, 4, 5 or 6. The results of the bear's study are below.



How many hexagons contain honey?

(A) 7

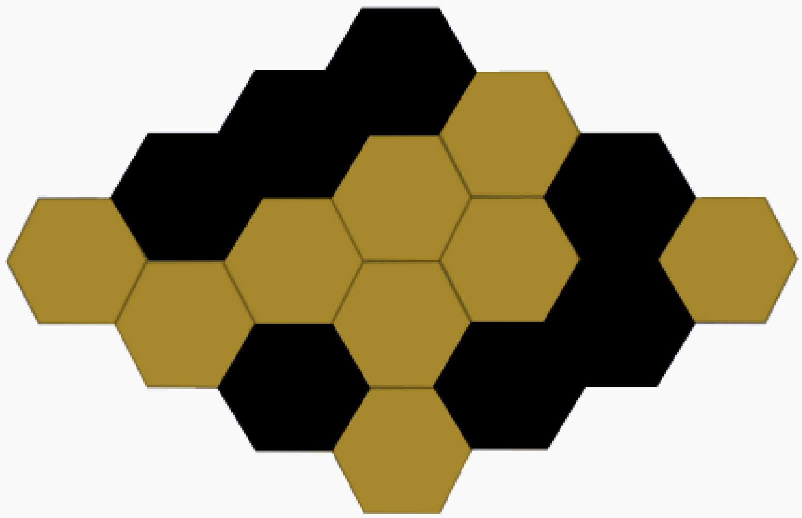
(B) 8

(C) 9

(D) 10

Solution

Starting from the right-most hexagon, we see that the two adjacent hexagons must not contain honey. Note that the number in a hexagon does not count whether itself contains honey but only whether its adjacent hexagons have honey. Following the logic, we get the following honeycomb, where a hexagon is black if it does not contain honey and brown if it contains honey.



We can count 9 hexagons that contain honey, so the answer must be (C).

Gauss Mathematics Contests

The Gauss Contests introduce students in Grades 7 and 8 to a broader perspective of mathematics in a fun, accessible way. Intriguing problems and a multiple-choice format make the Gauss Contests a wonderful opportunity for all students in these grades to grow their interest in and get curious about the power of math.

> **Note**
> There is a Gauss 7 and a Gauss 8 contest. Although they might share some questions, they are different contests from each other. Since you are probably younger than grade 7, you may choose to write either one of Gauss 7 or Gauss 8.

Audience

- All students in Grades 7 and 8
- Interested students from lower grades

Contest Date

- Ordering (Registration) Deadline: April 25, 2023
- Contest Date: May 17, 2023 (North & South America)

Format

- Number of questions: 25 multiple-choice questions
- Duration: 60 minutes
- Score: out of 150
- Format (delivering) of the contest: paper or online
- Some calculators permitted
- Paper dictionaries allowed

Calculating devices are allowed, provided that they do not have: internet access, the ability to communicate with other devices, information previously stored by students (such as formulas, programs, notes, etc.), a computer algebra system, dynamic geometry software.

Mathematical Content

Questions are based on curriculum common to all Canadian provinces. The last few questions are designed to test ingenuity and insight. Rather than testing content, most of the contest problems test logical thinking and mathematical problem-solving.

Recognition

- A Certificate of Participation is provided for each participant.
- A Certificate of Distinction is provided for each participant scoring in the top 25% of all participants within their own school, for schools with at least 4 participating students.
- A Certificate of Outstanding Achievement is provided to the highest achieving participant in their school on each of the Grade 7 and 8 Contests, for schools with at least 10 participating students.
- The names of some of the top-scoring participants among all those writing the contests are posted online.

Strategies

Some useful strategies when writing Gauss Contests:

1. Using the information given. When they give you any information, they are probably expecting you to use that information to solve the problem. Choosing which information to use and the order in which each piece information is used is also important when it comes to problem solving.
2. Working from the 5 possible answers. Remember, 1 out of 5 possible answers must be correct. You can look at the possible answers first, and rule out the ones that cannot be the correct answer.
3. Drawing/Using the diagram. Remember, diagrams are not drawn to scale. Hence, drawing your own diagram could be more helpful than using the one on the question. If the question does not provide a diagram, draw your own! Incorporating some information provided in the question on the diagram will be crucial to problem solving. For example, if they provide a length, indicate that on your diagram! Drawing some lines to further divide your diagram could help you in so many problems.
4. Looking for patterns. If they are asking you to find the 2023rd number in a list, they are NOT

expecting you to write down all numbers that come before. Always look for patterns and think about ways to apply that pattern to find your answer!

5. Working backward. Since you are given 5 possible answers, you can always try them one by one and find the one that makes sense.

Past Gauss Problems

1. The value of $(2+4+6)-(1+3+5)$ is

   (A) 0 (B) 3 (C) -3 (D) 21 (E) 111

Solution

$$\begin{aligned}(2+4+6)-(1+3+5) &= 12-9\\ &= 3\end{aligned}$$

So the answer is (B).

2. Michael has \$280 in \$20 bills. How many \$20 bills does he have?

   (A) 10 (B) 12 (C) 14 (D) 16 (E) 18

Solution

Since $\$280 \div \$20 = 14$, the answer is (C).

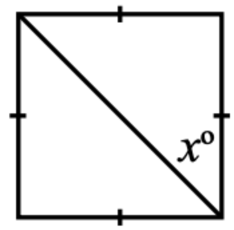
3. When two integers between 1 and 10 are multiplied, the result is 14. What is the sum of these two integers?

   (A) 2 (B) 5 (C) 7 (D) 9 (E) 33

Solution

The positive factors of 14 are 1, 2, 7, and 14. Since we can only take integers between 1 and 10, we know that the two integers are 2 and 7. Since $2+7=9$, the answer is (D).

4. In the square shown, x is equal to



(A) 0 (B) 45 (C) 60 (D) 180 (E) 360

Solution
A square has the property that each interior angle is 90°. Since the diagonal line creates a right-angled isosceles triangle, we can deduce that triangle has a 90° angle and two 45° angles. So $x°$ must be 45° and the answer is (B).

5. The sum of three consecutive integers is 153. The largest of these three integers is

(A) 52 (B) 50 (C) 53 (D) 54 (E) 51

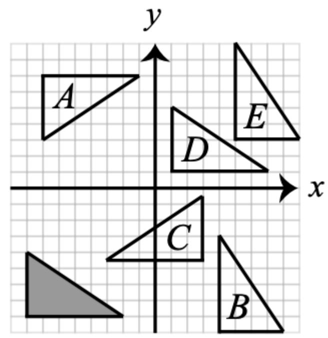
Solution
The average of 153 is $\frac{153}{3} = 51$. So 51 must be the middle number, and the largest number is 52. Therefore the answer is (A).

6. The number that goes into the $\square$ to make $\frac{3}{7} = \frac{\square}{63}$ true is

(A) 27 (B) 9 (C) 59 (D) 63 (E) 3

Solution
In order to find the missing number, we have to find how much the fraction on the right is scaled up by. Since $63 \div 7 = 9$, we can see that we multiplied the denominator of the left fraction by 9 to get the right fraction. So $3 \times 9 = 27$, and the answer is (A).

7. When the shaded triangle shown is translated, which of the following triangles can be obtained?

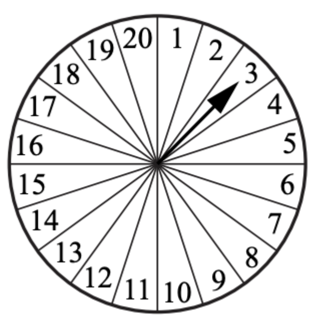


(A) A (B) B (C) C (D) D (E) E

Solution

A translation is simply moving a shape without rotating, flipping, or changing its size. The only triangle that is in the same orientation as the shaded triangle is D, so the answer is (D).

8. A circular spinner is divided into 20 equal sections, as shown. An arrow is attached to the centre of the spinner. The arrow is spun once. What is the probability that the arrow stops in a section containing a number that is a divisor of 20?

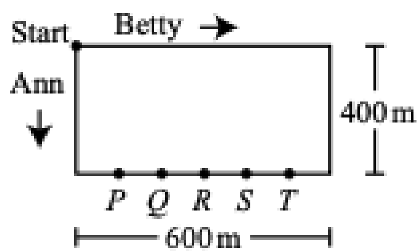


(A) $\frac{12}{20}$ (B) $\frac{14}{20}$ (C) $\frac{15}{20}$ (D) $\frac{7}{20}$ (E) $\frac{6}{20}$

Solution

The factors of a number will also be its divisors. Since the positive factors of 20 are 1, 2, 4, 5, 10, and 20, those are also all the positive divisors of 20. Therefore the answer is (E).

9. Betty and Ann are walking around a rectangular park with dimensions 600 m by 400 m, as shown. They both begin at the top left corner of the park and walk at constant but different speeds. Betty walks in a clockwise direction and Ann walks in a counterclockwise direction. Points P, Q, R, S, T divide the bottom edge of the park into six segments of equal length. When Betty and Ann meet for the first time, they are between Q and R. Which of the following could be the ratio of Betty's speed to Ann's speed?



(A) 5 : 3 (B) 9 : 4 (C) 11 : 6 (D) 12 : 5 (E) 17 : 7

Solution

Points P, Q, R, S, T divide the bottom edge of the park into six segments of equal length, each of which has length $600 \div 6 = 100$ m. If Betty and Ann had met for the first time at point Q, then Betty would have walked a total distance of $600 + 400 + 4 \times 100 = 1400$ m and Ann would have walked a total distance of $400 + 2 \times 100 = 600$ m.

When they meet, the time that Betty has been walking is equal to the time that Ann has been walking and so the ratio of Betty's speed to Ann's speed is equal to the ratio of the distance that Betty has walked to the distance that Ann has walked. That is, if they had met for the first time at point Q, the ratio of their speed's would be 1400 : 600 or 14 : 6 or 7 : 3.

Similarly, if Betty and Ann had met for the first time at point R, then Betty would have walked a total distance of $600 + 400 + 3 \times 100 = 1300$ m and Ann would have walked a total distance of $400 + 3 \times 100 = 700$ m. In this case, the ratio of their speed's would be 1300 : 700 or 13 : 7. When Betty and Ann actually meet for the first time, they are between Q and R.

Thus Betty has walked less distance than she would have had they met at Q and more distance than she would have had they met at R. That is, the ratio of Betty's speed to Ann's speed must be less than $7:3$ and greater than $13:7$. We must determine which of the five given answers is a ratio that is less than $7:3$ and greater than $13:7$. One way to do this is to convert each ratio into a mixed fraction.

That is, we must determine which of the five answers is less than $7:3=\frac{7}{3}=2\frac{1}{3}$ and greater than $13:7=\frac{13}{7}=1\frac{6}{7}$. Converting the answers, we get $\frac{5}{3}=1\frac{2}{3}$, $\frac{9}{4}=2\frac{1}{4}$, $\frac{11}{6}=1\frac{5}{6}$, $\frac{12}{5}=2\frac{2}{5}$, and $\frac{17}{7}=2\frac{3}{7}$. Of the five given answers, the only fraction that is less than $2\frac{1}{3}$ and greater than $1\frac{6}{7}$ is $2\frac{1}{4}$. If Betty and Ann meet for the first time between Q and R, then the ratio of Betty's speed to Ann's speed could be $9:4$.

10. In the six-digit number $1ABCDE$, each letter represents a digit. Given that $1ABCDE\times 3=ABCDE1$, the value of $A+B+C+D+E$ is

(A) 29 (B) 26 (C) 22 (D) 30 (E) 28

Solution

Note that A, B, C, D, and E must be integers between 0 and 9. Since the product's last digit is a 1, it must be that something times 3 equals a number with a 1 on the end. The only way to get this with integers between 0 and 9 is $3\times 7=21$. Substituting $E=7$, we get

$$\begin{array}{rcccccc} & 1 & A & B & C & D & 7 \\ \times & & & & & & 3 \\ \hline & A & B & C & D & 7 & 1 \end{array}$$

Since $7\times 3=21$, 2 is carried to the tens column. So, the units digit of $D\times 3+2$ is 7, and so the units digit of $D\times 3$ is 5. Therefore, the only possible value of D is 5. Substituting $D=5$, we get

$$\begin{array}{rcccccc} & 1 & A & B & C & 5 & 7 \\ \times & & & & & & 3 \\ \hline & A & B & C & 5 & 7 & 1 \end{array}$$

Since $5\times 3=15$, 1 is carried to the hundreds column. So, the units digit of $C\times 3+1$ is 5, and so the units digit of $C\times 3$ is 4. Therefore, the only possible value of C is 8. Substituting $C=8$, we get

$$\begin{array}{r} 1\ A\ B\ 8\ 5\ 7 \\ \times \qquad\qquad 3 \\ \hline A\ B\ 8\ 5\ 7\ 1 \end{array}$$

Since $8 \times 3 = 24$, 2 is carried to the thousands column. So, the units digit of $B \times 3 + 2$ is 8, and so the units digit of $B \times 3$ is 6. Therefore, the only possible value of B is 2. Substituting $B = 2$, we get

$$\begin{array}{r} 1\ A\ 2\ 8\ 5\ 7 \\ \times \qquad\qquad 3 \\ \hline A\ 2\ 8\ 5\ 7\ 1 \end{array}$$

Since $2 \times 3 = 6$, there is no carry to the ten thousands column. So, the units digit of $A \times 3$ is 2. Therefore, the only possible value of A is 4. Substituting $A = 4$, we get

$$\begin{array}{r} 1\ 4\ 2\ 8\ 5\ 7 \\ \times \qquad\qquad 3 \\ \hline 4\ 2\ 8\ 5\ 7\ 1 \end{array}$$

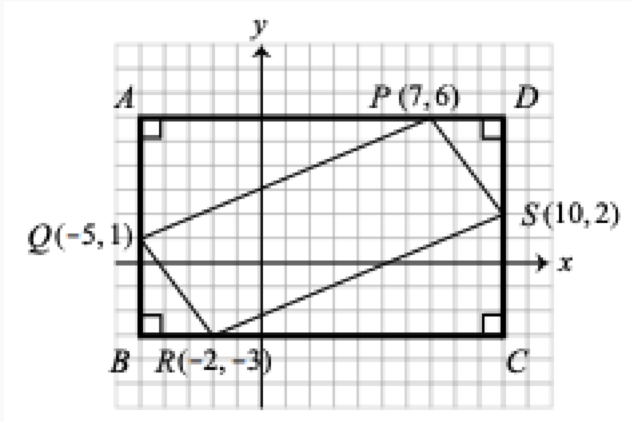
We can check to see that these are the correct digits, and we see that $A+B+C+D+E = 4+2+8+5+7+1 = 26$. So the answer is (B).

11. Four vertices of a quadrilateral are located at $(7, 6)$, $(-5, 1)$, $(-2, -3)$, and $(10, 2)$. The area of the quadrilateral in square units is

(A) 60 (B) 63 (C) 67 (D) 70 (E) 72

Solution

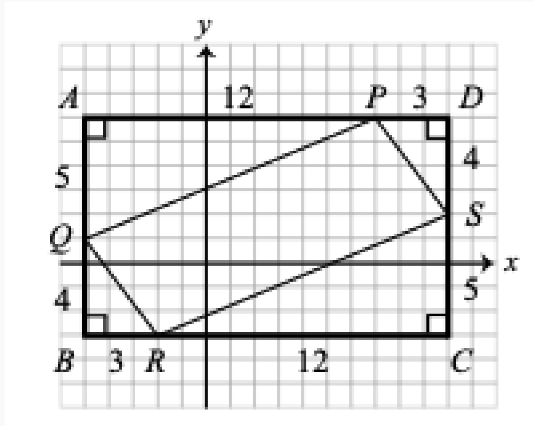
We begin by constructing rectangle $ABCD$ around the given quadrilateral $PQRS$, as shown. The vertical sides AB and DC pass through points Q and S, respectively. The horizontal sides AD and BC pass through points P and R, respectively. We determine the area of $PQRS$ by subtracting the areas of the four right- angled triangles, AQP, QBR, CSR, and SDP, from the area of $ABCD$.



To determine the horizontal side lengths of the right-angled triangles we count units along the x-axis, or we subtract the x-coordinates of two vertices. For example, since AB is vertical and passes through $Q(-5, 1)$, the x-coordinates of A and B are equal to that of Q, which is -5. Thus, the length of AP is determined by subtracting the x-coordinate of A from the x-coordinate of P, which is 7. Therefore the length of AP is $7 - (-5) = 12$. Similarly, the length of BR is $-2 - (-5) = 3$.

Since DC is vertical and passes through $S(10, 2)$, the x-coordinates of D and C are equal to that of S, which is 10. Thus, the length of PD is $10 - 7 = 3$, and the length of RC is 10 $- (-2) = 12$. To determine the vertical side lengths of the right-angled triangles we may count units along the y-axis, or we may subtract the y-coordinates of two vertices. For example, since AD is horizontal and passes through $P(7, 6)$, the y-coordinates of A and D are equal to that of P, which is 6. Thus, the length of AQ is determined by subtracting the y-coordinate of Q (which is 1) from the y-coordinate of A. Therefore the length of AQ is $6 - 1 = 5$. Similarly, the length of DS is $6 - 2 = 4$.

Since BC is horizontal and passes through $R(-2, -3)$, the y-coordinates of B and C are equal to that of R, which is -3. Thus, the length of QB is $1 - (-3) = 4$, and the length of SC is $2 - (-3) = 5$.



The area of $\triangle AQP$ is $\frac{1}{2} \times \text{AQ} \times \text{AP} = \frac{1}{2} \times 5 \times 12 = 30$.
The area of $\triangle CSR$ is also 30.
The area of $\triangle QBR$ is $\frac{1}{2} \times \text{QB} \times \text{BR} = \frac{1}{2} \times 4 \times 3 = 6$.
The area of $\triangle SDP$ is also 6.

Since $AB = AQ + QB = 5 + 4 = 9$ and $BC = BR + RC = 3 + 12 = 15$, the area of $ABCD$ is $9 \times 15 = 135$. Finally, the area of $PQRS$ is $135 - 30 \times 2 - 6 \times 2 = 135 - 60 - 12 = 63$.